

Proof: $\sqrt{x+2}\sqrt{x-1} + \sqrt{x-2}\sqrt{x-1} = 2$

let $v = \sqrt{x-1} \Rightarrow v \geq 0$

$\sqrt{v^2+1+2v} + \sqrt{v^2+1-2v} = 2$

$v+1 + |v-1| = 2$ note: the $|v-1|$ is there since

$v+1 + |v-1| = 2$

though $v \geq 0$, $v-1$ is not always positive, but principal square roots are positive and $(v-1)^2 = |v-1|^2$ for all v , so we put $|v-1|$

case 1, $v \geq 1$

$v+1 + v-1 = 2$

$2v = 2$

$v = 1$

case 2, $v < 1$

$v+1 - (v-1) = 2$

$1 - (-1) = 2 \quad 2 = 2$

$v \in [0, 1)$ since $v \geq 0$ as stated above and the ^{case is} $v < 1$

~~so $v=1 \Rightarrow \sqrt{x-1} = 1 \Rightarrow x=2$~~

~~$v \in [0, 1) \Rightarrow \sqrt{x-1} = 0 \Rightarrow x=1$~~

~~$\sqrt{x-1} = 1 \Rightarrow x=2$~~

~~so $x \in [1, 2]$~~

to prove this $\sqrt{x-1} = v$

$\Rightarrow x = v^2 + 1$

so $v=1$ (or $v \in [0, 1)$) $\Rightarrow v \in [0, 1]$ so $0 \leq v \leq 1$

so $x \in [0^2+1, 1^2+1] = [1, 2]$ since the function $f(v) = v^2+1$ is increasing.

So $x \in [1, 2]$ or rather All real x s.t. $1 \leq x \leq 2$